Common pitfalls in practical multi-objective optimization

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Sustainable End-to-end Design-operation

Goal

The literature on multi-objective optimization is vast. However, in practice not only the most commonly used approaches can yield bad quality solutions, but the noisy performance is often neglected and the problem is treated as deterministic. In this work we highlight some of the most crucial pitfalls when using common multi-objective algorithms in practice.

Motivation

- Multi-objective optimization problems are characterized by having more than a single optimal solution. In fact, often there are infinitely many equally optimal solutions. The Pareto-optimal set is then approximated with a subset of discrete solutions that reveal the essential trade-offs of the conflicting objectives.
- The literature offers a great variety of methods ranging from bio-inspired search paradigms, to sequential model-based optimization algorithms, to analytical methods. Most of these methods are non-trivial to be implemented in practice, and a number of issues are often over-looked. For example: noisy performance, multi-modal, non-linear and computationally expensive functions, handling of constraints, etc.).
- When using these techniques, certain caveats have to be considered in order for the methods to efficiently discover Pareto-optimal solutions.

Common solution approach

Scalarization methods

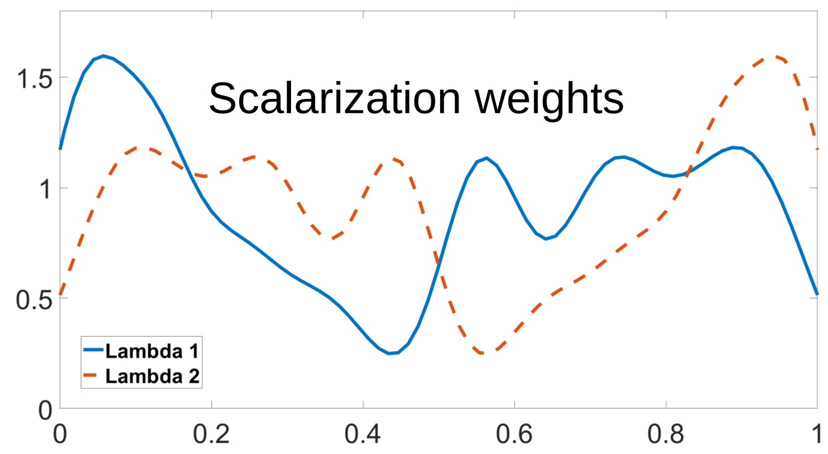
- Linear weighting scalarization

$$\min \sum_{i=1}^{m} \lambda_i f_i(\boldsymbol{x});\ \sum_{i=1}^{m} \lambda_i = 0$$

- Augmented Chebychev scalarization

$$\min \max_{i\in\{1,\dots,m\}} \lambda_i f_i(\boldsymbol{x}) + z\sum_{i=1}^{m} f_i(\boldsymbol{x})\,;\sum_{i=1}^{m} \lambda_i = 0$$

A *genetic algorithm* is often used as the base optimization method. The problem is then transformed into a (non)linear single-objective problem ------------->



Results

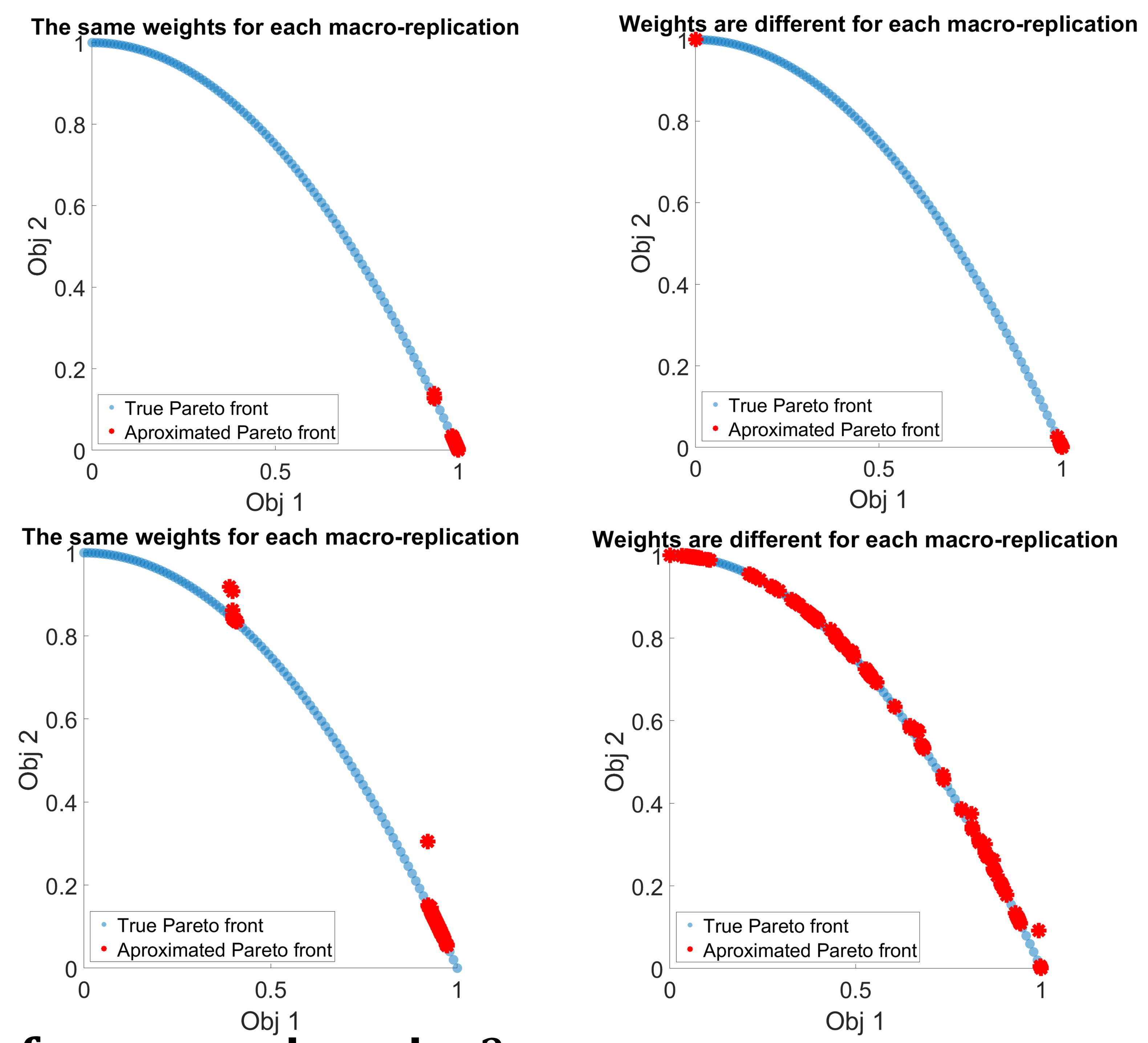
Experimental setting: a bi-objective problem is optimized using a population size of 50 and 100 generations. We perform 50 macro-replications using the same initial design. Scalarization weights are initially $\lambda_i = 0.5, i = \{1,2\}$ and $z = 0.05$.

Linear scalarization

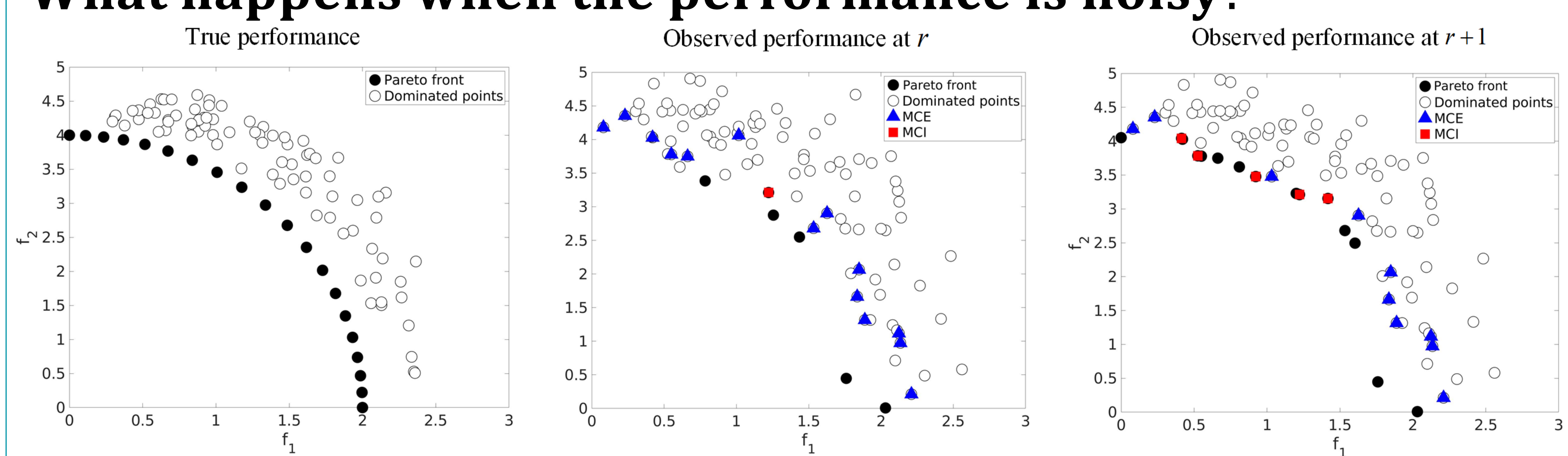
- Points in non-convex fronts may not be found
- Most solutions found are on the extremes of the PF
- Fixing or changing weights seems to have not a direct impact in the approximate fronts

Augmented Chebychev scalarization

- The z parameter ensures that weakly dominated points are avoided
- By changing the weights we are able to find all the points in the Pareto front



What happens when the performance is noisy?



We incur misclassification errors that change every time a new replication is allocated. We propose a method to mitigate this issue in [2].

	$m=2$	$m=3$	$m=5$	$m=7$	$m=9$
Total number of points	100	300	1000	1200	1500
Number of non-dominated points	20	200	500	700	900
Deterministic *HV*	0.3323	0.5585	0.8167	0.9172	0.9658
Noisy *HV*	0.4658	0.7020	0.8842	0.9418	0.3566
Deterministic *IGD*	0.0637	0.1844	0.8489	1.7357	3.0591
Noisy *IGD*	0.4136	0.4833	0.9108	1.7645	3.0121
Number of MCE errors	15	145	209	97	3
Number of MCI errors	4	28	303	415	493

- Standard quality metrics for deterministic multiobjective optimizers no longer work due to noise.
  - **Misclassification by exclusion (MCE):** We observe a point that is truly non-dominated as dominated.
  - **Misclassification by inclusion (MCI):** We observe a point that is truly dominated as non-dominated.
- The number of errors grows exponentially with growing number of objectives, and the values for standard metrics like HV and IGD are largely misleading.

Further reading

[1] *Emmerich, M.T.M., Deutz, A.H. A tutorial on multiobjective optimization: fundamentals and evolutionary methods. Nat Comput* ***17****, 585–609 (2018).*
[2] *Rojas* Gonzalez, S., Branke, J., & van Nieuwenhuyse, I. (2023). Multiobjective Ranking and Selection Using Stochastic Kriging. *arXiv preprint arXiv:2209.03919*.
[3] *Morales-Hernández, A., Rojas Gonzalez, S., Van Nieuwenhuyse, I., Jordens, J., Witters, M., & Van Doninck, B. (2023) "Expensive multi-objective optimization of adhesive bonding process in constrained settings". In Optimization and Learning: International Conference, OLA 2023 (to appear).*

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